Invited Lecture

The Mental Starters Assessment Project: Ambitious teaching in the South African context

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ABSTRACT In this paper, I detail the ways in which a South African initiative focused on mental mathematics in the early grades (the Mental Starters Assessment Project — MSAP) can be considered as an intervention aligned with the idea of ambitious instructional practice. In building this argument, I take note of the fact that the materials associated with the MSAP initiative are relatively prescriptive in their format, a feature that has sometimes been argued to work against the goals of ambitious instructional practice. The reasons for considering the MSAP an ambitious instructional practice initiative is linked in the paper with the attention given in the materials to working across the strands of mathematical proficiency, with local conditions and cultures driving the relatively prescriptive format of materials provision.

Keywords: Mental mathematics; Ambitious teaching; South Africa

1. Introduction

In this paper that follows from my ICME-14 Invited Lecture, I make an argument for why the mathematical content and format of the Mental Starters Assessment Project (MSAP) in South Africa can be seen — in context — as an intervention focused on the idea of 'ambitious mathematics teaching practice' (Lampert et al., 2010). Through the details that follow, I argue that this is the case even amidst a format that is relatively prescriptive about content and sequence, with these features being responsive to aspects of the early mathematics education context in the country.

The MSAP initiative, focused on early mental mathematics in South Africa, was rolled out as part of national policy in the opening mental starter section of the advocated mathematics lesson structure in Grade 3 in 2022. The MSAP model is a simple one: there are six mental mathematics units, each focused on a specific strategic mental skill, are taught as two units a term (with each unit taking three weeks of teaching) across the first three terms of the four-term year. Each unit also has a simple routinized structure: a 5 minute written pre-assessment that the teacher sets the class at the start of a unit (usually on a Monday morning) and then marks; eight starter activities — each made up of a quick warm-up task or tasks, a teacher led focus on two problems,

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and individual work on two or three similar problems — that are run at the start of lessons over the course of the interim three weeks; and a 5 minute written post-assessment that the teacher sets the class at the end of the unit (usually on a Friday morning) and again marks. Differences in marks at the individual and class level provide the teacher and children with a sense of the efficacy of the teaching and of improvements in learning related to the unit focus.

But underlying this simplicity, there have been six years of iterative design research that has distilled insights from earlier trials related to the content, format and sequence of mathematical tasks and to the teacher support materials offered alongside the student materials. In this paper, I detail the ways in which — within the simplicity of its model — the MSAP format includes attention to all of the strands of mathematical proficiency as outlined in the work of Kilpatrick, Swafford and Findell (2000). Attention to a holistic focus on mathematical proficiency is commonly invoked as one of the key hallmarks of ambitious instruction (Kazemi et al., 2009), but this is usually coupled with a pedagogic form focused on discussion-based environments, teacher as facilitator, and elicitation of student understandings. In the MSAP model, we have attended extensively to distill mathematical attention to the strands of mathematical proficiency through the provision of a programme of mental mathematics tasks, representations and activities. These are, however, couched within a pedagogic form that reflects much that has been written about the conditions and culture of South African primary classrooms, as predominantly authoritative and teacher-led instructional spaces, where gaps and fragilities in teachers' mathematical knowledge are widespread (Hoadley, 2018). Our approach to the MSAP initiative has therefore been focused on the provision of a programme of teaching materials that are seen as 'educative' (Schneider and Krajcik, 2002) in the sense that they are designed to support teachers' attention to teaching for meaning-making and progression in children's mental mathematics working in the terrain of these conditions.

In this paper, I begin by introducing the content and format of the MSAP materials before turning my attention to the ways in which these aspects provide attention to all of Kilpatrick et al.'s (2000) five strands of mathematical proficiency. I then discuss the reasons for a format which can be critiqued for being more prescriptive and more scripted in tone than some of the writing on ambitious instruction would advocate. In the concluding sections, I reflect on the nature of the balance between standardization of content/routines and responsive and flexible teaching in the MSAP materials in the national context, and the possibilities for educative materials in this particular format to bring about a change in the ground of mental mathematics and early number teaching and learning.

2. The MSAP Content and Format

The MSAP initiative came about in a collaboration between two research and development Chairs in South Africa (myself and my colleague, Mellony Graven) and our respective teams, the national Department of Basic Education (DBE), and partners across the education sector: key professional (Association of Mathematics Educators of South Africa — AMESA) and research (Southern African Association for Research in Mathematics, Science and Technology Education — SAARMSTE) organizations and the non-governmental organization sector (OLICO Youth). The DBE were keen to explore formative in-class assessment models for use in early grades' mathematics, and invited the two Chairs to look at options. In both projects, attention had been given in earlier research and development activities to supporting early number learning. The reasons for this focus were two-fold. Firstly, there was extensive evidence in South Africa of the widespread use of highly inefficient counting-based approaches to the four operations, well into the increasing number ranges that children are expected to work in as they progress through the primary grades (Schollar, 2008). These counting approaches are commonly seen in finger counting and in pages of 'tally' counts on paper in children's work. Second, number forms the single largest topic area in the mathematics curriculum in the early grades, making up more than 50% of the content distribution between Grades 1–3. Thus, substantively and pragmatically, improving early number learning leverages improvements in mathematics learning overall.

Mental mathematics is widely described in the literature as an important avenue for supporting the building of strong foundations to number working in the idea of number sense, but Beshuizen and Anghileri's (1998) writing points to ongoing differences in the emphasis accorded to mental mathematics in taught and assessed curricula across different countries. This left us with limited examples for models of integrating work on mental mathematics, and particularly so when thinking about the national systemic scale that the Chair projects were devised to attend to. We were influenced by the writing of, and our interactions with, two international experts: Mike Askew and Bob Wright, both of whom had paid attention over an extended period of time to how moves beyond calculating-by-counting could be encouraged. Specifically, we became interested in Askew et al.'s (1997) attention to tasks that emphasized the need for reasoning about number relationships — e.g.: How does knowing that double 16 is 32 help us to deal with $16 + 17$? Wright, Ellemor-Collins and Tabor's (2012) writing on progression in early number learning was also influential in helping us to consider task sequences and representations that helped to emphasize the idea of 'base ten thinking': using the structure of the decimal system to identify and work with multiples of 10 as friendly numbers and relationships between numbers and multiples of 10 as benchmarks to use for the purpose of efficient calculations.

We also worked with the South African Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011) for the early grades in South Africa, which made recurring reference to mental mathematics, but was coupled with an assessment regime that tended to sideline focus on efficient ways of working by marking simply for correct answers without attention to whether these were produced by efficient working with number relationships or by rudimentary counting in ones (Graven and Venkat, 2021). Working with the aspects that were mentioned in the CAPS document (DBE,

2011) in conjunction with the literature on early mental mathematics, we identified six foci for units, presented and exemplified in our final MSAP Teacher Guide document (Graven et al., 2021) as shown in Fig. 1.

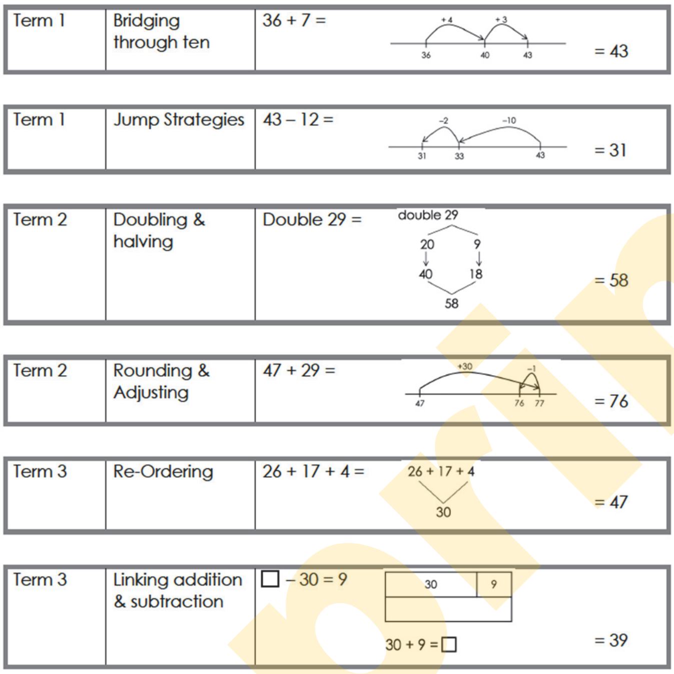


Fig. 1. The six mental mathematics strategy units in the MSAP

The three-week model for each unit (short time-limited pre-assessment, eight lesson starter activities, short time-limited post-assessment) has already been outlined above. In thinking about *how* to focus on mental mathematics, we were mindful of a ground in which there was evidence of very limited attention to the need to establish and grow a bank of basic established results. Venkat and Naidoo’s (2012) writing had drawn attention to the ways in which repeated instruction to children to use concrete resources to count in order to calculate answers sidelined attention to answers produced previously, resulting in a continuous cycle of ‘first principles’ working. Gaps in early grades’ teachers’ understanding of the importance of early number progression were reflected in these instructional approaches, and pointed us to the need to explicate aspects that — at Grade 3 level — children could, in relation to CAPS content, be expected to work with at the level of near automaticity. The strategic focus of each unit was studied and decomposed mathematically for the range of underlying ‘fluencies’ required to work in efficient ways. In some cases, the list of fluencies was edited or expanded based on empirical trialing (see Graven and Venkat, 2021 for more detail on the trials and their outcomes). By way of example, the list of underlying fluencies for working on Jump Strategies that we ended up with consisted of the following aspects:

- count on or back in 10s from any number (e.g. 12, 22, 32, or 57, 47, 37, ...)
- add or subtract 10 from any number (e.g. $43 + 10 = 53$ or $89 - 10 = 79$)
- add a multiple of ten to any number (e.g. $61 + 20 = 81$)
- subtract a multiple of ten from any number (e.g. $46 - 30 = 16$)
- jumping to the next multiple of ten after a number (e.g. $32 \rightarrow 40$)
- jumping to the multiple of ten before a number (e.g. $56 \rightarrow 50$)

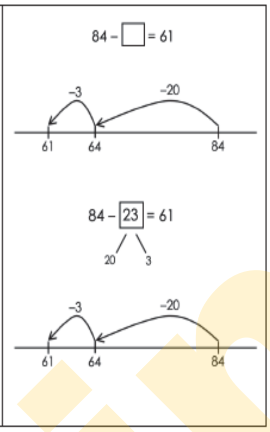
In wanting to communicate with teachers through the materials in a language that they would be familiar with, we described fluency-oriented tasks as focused on ‘**Rapid Recall**’, and incorporated a 1-minute warm up task sequence into each lesson starter for every unit focused on consistent attention to developing these fluencies in a kick-off whole-class ‘Warm-up’ activity segment.

As noted already, these fluencies were necessary to support children to become more successful with using the strategy in focus in each unit in their calculation, rather than reverting to the unit counting that was so prevalent on the ground. The core tasks in each lesson starter were then focused on ‘**Strategic Calculating**’ — calculating using the focal strategy. In the case of the Jump Strategy unit for example, this involved working with two-digit addition and subtraction tasks through the use of what Beishuizen (1993) calls N10 strategies: where the first number is kept whole and the second number is broken down into its place value decomposition for easier mental addition or subtraction. An extensive evidence base points to this strategy proving particularly useful in surmounting the common errors seen in the widely used column algorithms when ‘carrying’ in addition and ‘borrowing’ in subtraction become necessary. In a carefully graded sequence of starter activities, the complexity of tasks is gradually expanded to include examples that incorporate bridging through ten steps within the use of the jump strategy, and also examples that include missing addend/subtrahend tasks, in which the tens and ones jumps have to be ‘built up’ to find the missing number. There are openings here for conversations about how ‘building up’ numbers using their place value decompositions and ‘breaking down’ numbers into their place value decompositions are related, and as such, tasks like these represent opportunities for focusing on mathematical practices such as ‘doing and undoing’ (Mason, 1988) that is a central and recurrent idea in mathematics. In the South African context where problems with coherent instructional explanations have been widely discussed across all phases, the MSAP materials include illustrations of the instructional talk that can accompany one of the tasks in each starter activity. This is detailed in text for teachers in the Teacher Guide document (Graven et al., 2021a) with a ‘talking hands’ video clip included alongside (a feature included for all the lesson starters) as illustrated in Fig. 2 on the next page.

The tasks in Fig. 2 illustrate also our inclusion of key representations that have been identified as supporting increasingly efficient mental calculation 1–2 the empty number line that has an extensive evidence base for its efficacy in studies located in

Task Sequence

In this lesson we use jump strategies to solve missing number problems.

Problem: 84 – ☐ = 61 Plot ‘84’ on the number line. Teacher: We need to jump back to 61. Mark ‘61’ on the number line. Teacher: What tens jump and what units jump should we make? Learners: Minus 20 to get to 64 and minus 3 to get to 61 Draw these jumps on the number line. Ask: How much did we jump back in total? Learner: 23 Write the answer in the block. Teacher: So 84 – 20 – 3 is the same as 84 – 23 = 61 Write number sentence as shown.	

https://youtu.be/BHC9jDIUdRI

Fig.2. Jump Strategy Lesson Starter 8 (Graven et al., 2021b)

the Realistic Mathematics Education (RME) (van den Heuvel-Panhuizen, 2000). We also included recurring reference to part-part-whole models as these have been identified as important and useful for sorting out the ways in which given quantities in a problem are related to each other (e.g. Murata, 2008) with Xin’s (2012) work illustrating their particular usefulness and importance for students falling behind the mainstream.

In many ways, moderate aims would have suggested that we stop with these two goals for each unit: improving Rapid Recall and Strategic Calculating. However, we were sensitive to the evidence that improving children’s ability to carry out calculations more efficiently could leave aside attention in teaching to a focus on the structural relations that underpin efficient strategies in mental calculation (e.g. Polotskaia and Savard, 2018). Open number sentence formats have been presented in earlier research as one avenue for drawing attention to the representation of structural relationships. An example from the work of Hopkins, Russo and Downton (2019) illustrates items that are focused on the number relationship underlying strategic calculation, rather than on the calculation itself:

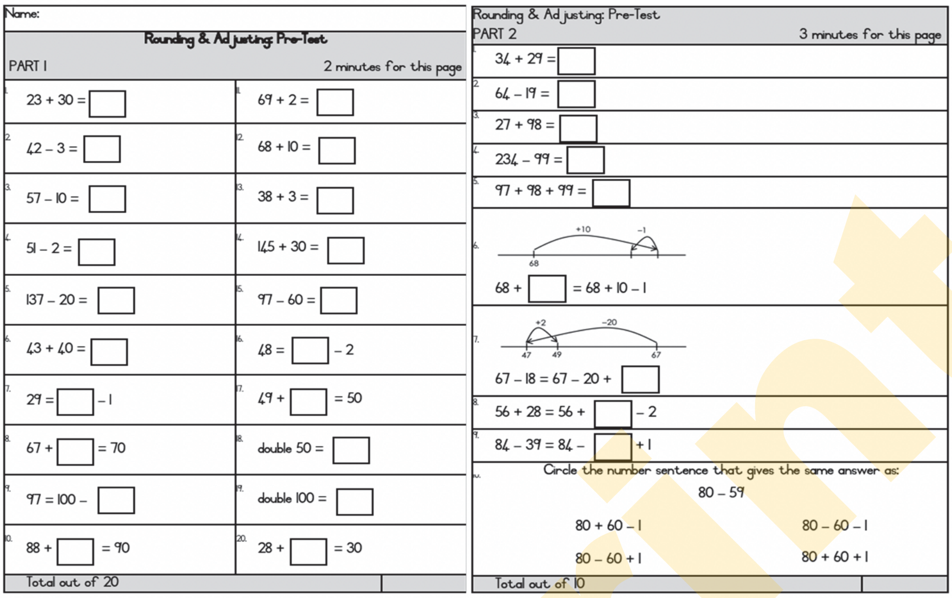
$$11 + 3 + 9 = \_ + 3$$

In this example, the strategic calculation skill in focus is reordering, and draws on the associativity property of addition. We referred to items focused on structural relations within the aim of **strategic thinking**, to emphasize that these kinds of questions did not involve calculating. In the MSAP materials, references to strategic thinking included recurring reference to the use of the key representations identified in the excerpts above — part-part-whole and empty number line models — that were used in the context of the rapid recall and strategic calculating tasks.

The pre- and post-assessments linked with each unit included items across all three categories: rapid recall, strategic calculating and strategic thinking. Hopkins et al. (ibid.) note that much of the work focused on strategic efficiency in early number working has been drawn from in-depth one-on-one work with children. While these studies have provided usefully rich illustrations of progression in early number working, the approaches tend to be impractical for larger-scale development activity that includes assessment components. In order to communicate the message about the need for efficient working and an understanding of underlying structure for teaching and assessment at systemic level, we needed a relatively simple model that could be communicated succinctly with early grades' district Subject Advisers nationally who we could provide training for to support policy implementation. Further, the materials had to be suitable for conditions on the ground of large classes and limited classroom resources beyond pencil and paper. The decision we came to on this was to design pre- and post-assessments as double-sided single page time-limited tests for learners. The front side of the test, across all units, features 20 items focused on rapid recall. Children are told they have 2 minutes to answer as many of the questions as they can, with the teacher telling them when to stop writing. Following this, children are asked to turn the test over to the second side, with this page – again across all units, containing 10 items drawn from across the strategic calculating and strategic thinking categories linked to the focal unit. Children are given 3 minutes to complete as many items on the second page as they can. The two pages from the Rounding and Adjusting unit pre-assessment are shown in Fig. 3 (on the next page).

The time-limited format provided a mechanism for communicating the importance of increasingly efficient working, with the low-stakes in-class assessment model allowing us to emphasize that what was important was individual pre- to post-test improvement rather than comparative performance either within or between classes and schools. Stott and Graven's (2013) earlier work had shown that this emphasis had mitigated children's anxiety about the time-limited format, with excitement rather than fear predominating in children's work.

I go on now to outline the strands of mathematical proficiency, and to discuss the ways in which the mathematical content and its packaging in the MSAP model was designed to address these strands.



Name:

Rounding & Adjusting Pre-Test

PART 1 — 2 minutes for this page

1. 23 + 30 = □	11. 69 + 2 = □
2. 42 – 3 = □	12. 68 + 10 = □
3. 57 – 10 = □	13. 38 + 3 = □
4. 51 – 2 = □	14. 145 + 30 = □
5. 137 – 20 = □	15. 97 – 60 = □
6. 43 + 40 = □	16. 48 = □ – 2
7. 29 = □ – 1	17. 49 + □ = 50
8. 67 + □ = 70	18. double 50 = □
9. 97 = 100 – □	19. double 100 = □
10. 88 + □ = 90	20. 28 + □ = 30
Total out of 20	

Rounding & Adjusting: Pre-Test

PART 2 — 3 minutes for this page

1. 34 + 29 = □
2. 64 – 19 = □
3. 27 + 98 = □
4. 234 – 99 = □
5. 97 + 98 + 99 = □
6. +10, –1, 68
68 + □ = 68 + 10 – 1
7. +2, –20, 47, 49, 67
67 – 18 = 67 – 20 + □
8. 56 + 28 = 56 + □ – 2
9. 84 – 39 = 84 – □ + 1
10. Circle the number sentence that gives the same answer as:
80 – 59
80 + 60 – 1 | 80 – 60 – 1
80 – 60 + 1 | 80 + 60 + 1

Total out of 10

Fig. 3. Rounding and adjusting pre-assessment

3. Addressing Mathematical Proficiency

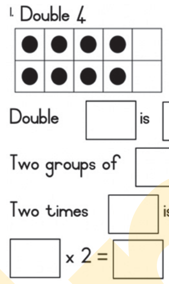
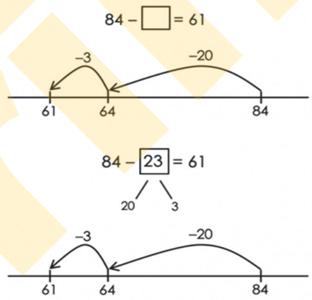
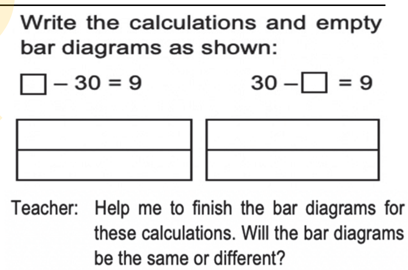
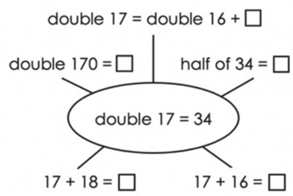
3.1. Outlining the strands

Kilpatrick et al. (2001) proposed the idea of mathematical proficiency as the output, or consequence, of taking on: “a composite, comprehensive view of successful mathematics learning” (p.116). The intertwined strands that they viewed as critical to achieving mathematical proficiency are: conceptual understanding; procedural fluency; strategic competence; adaptive reasoning; and productive disposition. The book: Adding it Up, offers extensive illustration and discussion on each of the strands, so I do not repeat that detail here. Instead, I offer a small number of quotes from this work that offer a sense of the focus of each strand (see Tab. 1 on the next page). This is followed by advice, examples and assessment items drawn from the MSAP materials and assessments that exemplify some of the ways in which the strand was addressed in the project. Key features of these examples are discussed and elaborated below.

The exemplifications from the MSAP materials point to attention to all of the strands in the content and format of the intervention. In this analysis, I use writing on each of the strands to delve into some of the ways in which the strands are incorporated into the materials, and to note if there are specific emphases within the ways in which the strand is presented.

As the quote above makes clear, a key marker of conceptual understanding is fluent moves between representations. In the exemplar in Tab. 1, taken from the Doubling and Halving unit, I note the attention to translating — not just between concrete and symbolic representations of a double number — but also to different ways of speaking

Tab. 1. Descriptions of the focus of each of the five strands of mathematical proficiency and exemplifications of strands from MSAP materials

Strand	Descriptive quotes	Exemplification from MSAP materials
Conceptual understanding	'being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes' p.119	
Procedural fluency	'knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently' p.121	
Strategic competence	'the ability to formulate mathematical problems, represent them and solve them' (p.124)	
Adaptive competence	'the capacity to think logically about the relationships among concepts and situations' (p.129)	
Productive disposition	habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (p116) 'the tendency … to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics' (p.131)	'Each unit begins and ends with a short test for the learners. Marking these tests provides information for you and the learners about how much they have improved in using that particular set of skills during the three weeks' (p. iv)

about doubles in everyday language. Variation in representations was built in across all units, with emphasis on translating between these — for example, translating the information in a missing number sentence like $4 + \_ = 23$ into a part-part-whole diagram, and then translating from the part-part-whole diagram into the range of number sentences that could be attached to this structure. Underlying our attention to building in variation, there was writing in variation theory that viewed moves between representations as a key dimension of variation to incorporate (Watson and Mason, 2005) for building flexible conceptual understanding, as well as indications — in a context of limitations in teachers' use of coherent and connected language (Mathews, 2021) — to emphasize variations in language connected to core ideas within a concept in order to familiarize learners with the different forms. Additionally, Gu, Huang and Gu's (2017) noting that careful variation provided useful instructional scaffolding in the context of large classes and predominantly teacher-led instructional settings offered a strong cultural fit with South African classroom settings

Procedural fluency was important in the South African context given the extensive evidence showing that many children were unable to carry out basic procedures in efficient ways drawing on a bank of known facts. Thus, we needed to communicate the importance of retention of a growing bank of established facts, and also to illustrate how this bank could be used in effective and efficient calculation procedures. In the MSAP model, procedural fluency was incorporated in the attention to a range of underlying basic fluencies in the warm up section of the starter that are then brought together in the execution of efficient calculation procedures linked to a variety of focal strategic calculations for each unit. In the Jump Strategies exemplar in Tab. 1, finding the missing addend using this approach involves fluency with subtracting a multiple of ten and rapid recall of the difference between 64 and 61 in the initial steps, as well as fluency with combining these place value-based decomposed parts into their numerical whole. All of these underlying fluencies are built into the warm up sections of the Jump Strategies unit and then drawn on in the teaching of the focal strategic calculations across the eight lesson starters.

In Kilpatrick et al.'s (2001) writing, strategic competence extends beyond the strategic calculation that was a key feature of the MSAP units. Strategic competence in the Kilpatrick et al. formulation includes attention to formulating and representing situations as well as solving problems. While the MSAP model made strategic calculating a key goal, and while the focus on mental mathematics meant situational or context-oriented problem-solving featured to a more limited extent, there were examples — as illustrated in Tab. 1 — of the need to consider how a given situation could be represented in an alternative form. In particular, given the international evidence of children finding it harder to solve missing start and missing addend/missing subtrahend problems in comparison to missing 'result' problems (Carpenter et al., 2000), we incorporated attention - again using ideas of variation and invariance - to supporting teachers and children to focus on the ways in which different number sentences involving the same numbers but in varying relationships implied different part-whole structures. Sorting out the part-whole representation in each case, with discussion of the rationales for deciding how to translate from each number sentence to a part-whole diagram exemplifies a key avenue that was used in the MSAP that expanded our attention to strategic competence beyond the focus on strategic calculation.

Adaptive reasoning has been written about widely as a central feature of mental mathematics underpinned by a strong number sense. Baroody's extended sequence of studies (among these Baroody, 1993; 2003) provide particularly salient evidence of what is gained in what he describes as the 'number sense' approach, in which interconnections between results from the primary focus of attention, rather than getting the answer to individual problems. These connections are promoted in the MSAP materials through two key avenues. Firstly, there is consistent and recurring attention to using known results to derive further results. The illustrative example in Tab. 1 provides a direct case of the use of this kind of approach, with the given result (double $17 = 34$) forming the start point for discussing and devising an expanding network of other results connected with this result. A second route through which adaptive expertise is promoted is through the attention to strategic thinking items in the starters and in the assessments. In these items, the focus is on different ways of expressing the structural relation in focus in the given task in ways that link to the focal strategy. For example, in the Bridging through Ten unit, the assessment includes items such as: $98 + 56 = 98 + 2 + \_$. Here, the student is invited to work with how the left hand side expression needs to be adapted to maintain equivalence with a specific bridging through ten action — in this case — coming into play.

Given the South African evidence of examples being treated highly 'separately' and with a repeated reversion to first principles counting strategies in early number, the focus within adaptive reasoning on connecting ideas and on leveraging connections to grow the base of known results through constructing further results derived from these, was particularly important.

Finally, and again as exemplified in Tab. 1, the messaging throughout the MSAP booklet is focused on individual learning and improvement, with the emphasis on low-stakes assessments geared towards in-class, developmental use by the class teacher. This messaging was important in a ground where previous early grades assessments (the Annual National Assessments) had been high-stakes for teachers and schools, and which the teacher unions had vociferously opposed and brought to a halt in 2015. The request to teachers to reinforce messages about looking for pre- to post assessment improvement, rather than inter-learner comparisons (or inter-class and inter-school comparisons) has been useful and important for communicating the need for consistent student working with the materials in order to become more skillful and efficient with the focal strategies over time.

Connecting all the strands, the work within the Realistic Mathematics Education (RME) tradition on the use of structured representations that were open to emergent working using increasingly efficient and sophisticated strategies was useful to our thinking. We followed the advice of Askew and Brown (2003) to judiciously select key representations that offered diagrammatic attention to number structure in ways that could underpin subsequent work with number in symbolic forms. The empty number line and part-part-whole models were key to this, and consistently connected with a range of symbolic forms, once again aimed at building connections. These models also provided, as RME advocates, important intermediary devices that function initially as diagrammed models of problem situations that are shared and discussed in class, with advice to teachers that these should, over time, become internalized mental models that function as tools to think with in students' mental mathematical working. Further, these structured models are closely linked with van den Heuvel-Panhuizen's (2008) calculating-by-structuring stage, which forms the key stage that follows the calculation-by-counting stage that South African evidence suggests that students and teachers struggle to transcend.

4. MSAP — Ambitious Instruction?

Lampert et al. (2010) cite Leinhardt and Greeno's (1986) work on the support offered by routines in supporting ambitious instruction. Specifically, the earlier work notes that routines represent ways in which to manage some elements of the background efficiently, allowing the foreground to be occupied by the mathematical goals at hand. This foregrounding of the mathematical goals has a particular urgency in the South African context, given the highly rudimentary strategies being used by children for early number problem-solving, but given too, the evidence of gaps in teachers' mathematical knowledge of how to push for progression in children's approaches. This is coupled with the evidence of teacher-led instruction. This raises questions about what to balance across the prescription-responsive freedom continuum if mental mathematics founded upon number sense is to be supported. Routines in the MSAP are built into a highly consistent structure for each unit and for the activity segments in each starter - with the aim that attention to basic fluencies and to teaching for strategic problem-solving start to become part of the 'natural' background of teachers' repertoires of practice. There is also recurrent reference to the empty number line and part-part-whole diagrams as tools for working with and then thinking with - again with a view to helping teachers to see these models as part of the basic toolkit for early mental mathematical working. The lesson starter activities themselves have a relatively prescribed core, but within these activities, the materials draw attention to the need to be responsive to early finishers with more complex tasks that teachers can devise, and to variations that may be seen in children's responses. Thus, while there is more detailing of the content of instructional explaining than is typical in Lampert et al.'s

(2010) work, there is room here also for working in ways that are more responsive and more tailored to the needs of the children in particular classes. Our decisions to work in this way are also responsive to the evidence that for education systems in developing country contexts with relatively low levels of performance, more prescription may be necessary in the introductory stages of bringing a system into functionality, and - importantly - there is evidence that this greater prescription is welcomed by teachers (Fleisch et al., 2016).

The outcomes from the early cycles of trials of the MSAP materials reflect Fleisch et al.'s position: that teachers find the materials use-able and useful. We also have evidence that using the materials can produce learning gains (Graven and Venkat, 2021). Thus, taken together, there is therefore the sense of materials in the MSAP package that can be considered both educative for teachers, and ambitious in terms of the mental mathematics learning that they seek to support.

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